Start Times Matricial Estimation with Stochastic Behavior for Periodic Real Time Tasks into Concurrent Systems

J. J. Medel [3,2], P. Guevara López [1,2] , F. Ríos Suriano [2,3]

[1] DGCH, CICATA- I. P. N. [2] CICATA-I. P. N., [3] UPIITA, – I. P. N.,

Tel.57296000 Ext. 50427, Ext. 64344

(pguevara, jjmedel) @ipn.mx, frios99@yahoo.com

Abstract. A multivariable stochastic dynamic model describing start times respect to real-time tasks with bounded properties: Stationary conditions, first order, with jitter and external perturbations bounded with a normal distribution without correlation that closely represent periodical behavior of real time tasks. To bear closer the task model in a concurrent system, internal dynamics description, and the parameter matrix in regressive model with oscillations used as multivariable estimator. Results of an example performed on a real-time platform presented, considering periodic and concurrent tasks, an instrumental variable algorithm is a basic tool in this kind of systems because it is a good estimator with convergence probability sense and its relatively easy implementation.

Keywords: Start time, estimation, real-time, periodic tasks, instrumental variable.

Computer Engineering.
Research in Computing Science 30, 2007, pp. 139- 145

1 Introduction

Periodic tasks are usually found in several applications like airplanes and control process where uniform monitoring is required. Modeling them is not simple because each of those needs an adequate representation. A set of periodic concurrent tasks represented as multivariable state model where in explicit way the model is function of the internal dynamic and previous states of the system's output information.

These matrix and perturbations are going to give the start times characteristics through system evolution. A parameter matrix estimator needed in order to adjust the model for reconstruction, tracing and prediction in real time.

2 Start Times Model in PRTT (Periodic Real Time Tasks)

As seen in [1], [4], [5], [6], [10], [14] and [15], PRTT model is represented by a stochastic, stationary, first order and first grade type difference equation; considering that external processor perturbations without correlation and obeying a normal distribution function.

Proposition 1. (Absolute Arrival Time for RTT). Into instance set, is the absolute arrival time vector L_k with index k described as:

$$L_k = L_{k-1} + \Pi_k . \tag{1}$$

Proposition 2. (Inter-arrival Time for RTT). Into instance set, is the Inter-arrival time vector Π_k with index k described as:

$$\Pi_k = A_k(\Pi_{k-1} - W_{k-1}) + U_k + W_k \tag{2}$$

Where: *A_k is the system parameter matrix with unknown dynamics but its bounded in agreement with [7], [11], [12] y [14]; Π_k is the Inter-arrival times vector of instances with index k; W_k is the external processor perturbations vector, represented through random variables with Gaussian distribution; U_k is the Inter-arrival time vector reference.*

Proposition 3. (Start Times for RTT). The starting times vector S_k of a feasible task with index k has a recursive description as:

$$S_k = S_{k-1} + \Pi_k + V_k - V_{k-1} . \tag{3}$$

Comment 1. If an inter-start times vector modeled as Π'_k require adding the *jitter* vector knowing it as V_k, justifying it as internal perturbation to the state equation such that:

$$\Pi'_k = X'_k + W_k, \tag{4}$$

$$X'_k = A_k X'_{k-1} + U_k + V_k, \tag{5}$$

Considering to (5) into (4), the inter-start times vector model is:

$$\Pi'_k = A_k(\Pi'_{k-1} - W_{k-1}) + U_k + W_k + V_k. \tag{6}$$

Proposition 4. (Periodic Tasks in Real Time). A PRTT set of J_i all its instances have inter-arrival times vectors Π_k round off to a periodic vector T_k.

Para A_k $\{a_{i,j,k}\} \subset [0,1]$ $a_{i,j,k}$ constante $\forall\ i,j,k \in Z^+$

Para U_k $\{u_{i,k} = T_i - a_{i,k} T_j\}$ $u_{i,k}$ constant $\forall\ ilk \in Z^+$

3 Real Time Parameter estimator (RTPE)

In order to trace the parameter matrix evolution into concurrent PRTT, the Real Time Estimator (RTPE) in agreement to [3], [7],[10], [14], [15], and [16] has a basic description:

Definition 1. (Real Time Parameter Estimator RTPE). All RTPE is a digital filter with the following basic conditions:

a. To Extract and to emit observable information (input and outputs observable signals, where $\{u(k)_i \in U(k)\}$ and $\{y(k)_j \in Y(k)\}$, respectively , with $i, j, k \in \mathbf{Z}^+$), in the sense considered by [2], [3], [7], [8], [11], [12] and [13],

b. To give correct answers respect to the process considered in some established criteria illustrated and conceptually described in [2],[11], [12] and [13],

c. Expressing the model filter in recursive sense (see [3], [7], [10], [13], [17]),

d. Convergence value bounded, respect to perturbation variance.

e. Matrix operation bounding the process dynamics restrictions.

Proposition 5. (Convergence in all Multivariables RTPE). All RTPE as a parameter estimator has an error functional bounded in probability sense (to see: [2],[7], [9], [16] and [17]), such that :

$$m^* = \arg\min \inf_{k \geq m} P\{|\hat{a}_k - a| \leq \Delta\} = 1. \tag{7}$$

Where Δ is the error limit, defined by noise variance; m is the convergence interval and m^* is the intervals set where the RTPE has a convergence rate in probability sense.

4 Estimation of Periodic Real Time Task Start Times with stochastic jitter using the Instrumental Variable technique.

As an example, we considered a model of concurrent PRTT set, described in (6): The relative arrival time model (Π_k) used as the observable signal, with the stochastic properties:

$$E\{W_{k+1}\Pi_k^T\}=[0],\quad E\{W_k\Pi_k^T\}=\Theta_{w_k}^2,\quad E\{V_k\Pi_k^T\}=\Theta_{v_k}^2,$$
$$E\{W_k(W_k)^T\}=\Theta_{w_k}^2,\quad E\{V_k(V_k)^T\}=\Theta_{v_k}^2,\quad E\{V_k(W_k)^T\}=[0]. \tag{8}$$

The estimator in agreement to [9] and obeying the properties expressed in Definition 1., has the form:

$$\hat{A}_k:=(\hat{A}_{k-1}B_{k-1}+\Pi_k Z_k^T)B_k^{-1}. \tag{9}$$

The estimation error in agreement to [2], [3], [7] and [9] is:

$$\Delta_k=\left|\hat{A}_k-A\right|. \tag{10}$$

Recursively, and in agreement to [9] the estimation error is:

$$\Delta_k:=((-A\Theta_{w_k}^2+\Theta_{v_k}^2)(I-A^2))(\Theta_{w_k}^2(2A^2+I+A^2)+\Theta_{v_k}^2(I+2A))^{-1}, \tag{11}$$

As the functional error in accordance with [9] and [15], expressed by the second probability moment in vector sense:

$$J_k=E(\Delta_k(\Delta_k)^T). \tag{12}$$

And considering its recursive description:

$$J_k=\tfrac{1}{k}(\Delta_k\Delta_k^T+(k-1)J_{k-1}). \tag{13}$$

The following data considered as hypothetical results evolution using the model expressed in (6):

$$A=\begin{bmatrix}0.5 & 0.2\\ 0.3 & 0.4\end{bmatrix},\quad \Theta_{v_k}^2=\begin{bmatrix}0.95 & 0.93\\ 0.97 & 0.94\end{bmatrix},\quad \Theta_{w_k}^2=\begin{bmatrix}0.95 & 0.93\\ 0.97 & 0.94\end{bmatrix}. \tag{14}$$

For the experimental implementation of the RTPE the following considered:

a. The maximal system deadline $\mathbf{D}_{k,\,max}$, is equal to period $\mathbf{T}_k$.
b. The Start time ($\mathbf{S}_k$- $\mathbf{L}_k$) was obtained such that $\mathbf{S}_k=\mathbf{L}_k+0.0015$ ms.
c. The sampling period T_k (Real Time temporizator impulse) for task activation is 10 ms.
d. Minimal deadline $\mathbf{D}_{k\,min}=1$ ms.
e. Convergence deadline of the PRTT set is: $d=4$ s.

For RTPE experiment (Fig. 1) obtained us the following results:

$m_{max}=354$ intervals, $t_{c\_max}=3{,}54$ s. The convergence time is $t_c=3.54$ s.

a. $m=$ [354, 213, 241, 318] intervals,
b. $t_c=$ [3.54, 2.13, 2.41, 3.18] s,
c. $d=$ 4 s. (400 intervals).

Finally, we considered the estimation algorithm expressed in (9). The Fig.1 and Fig. 2 are just examples of a RTPE behavior used as a parameter estimator trough the instrumental variable technique.

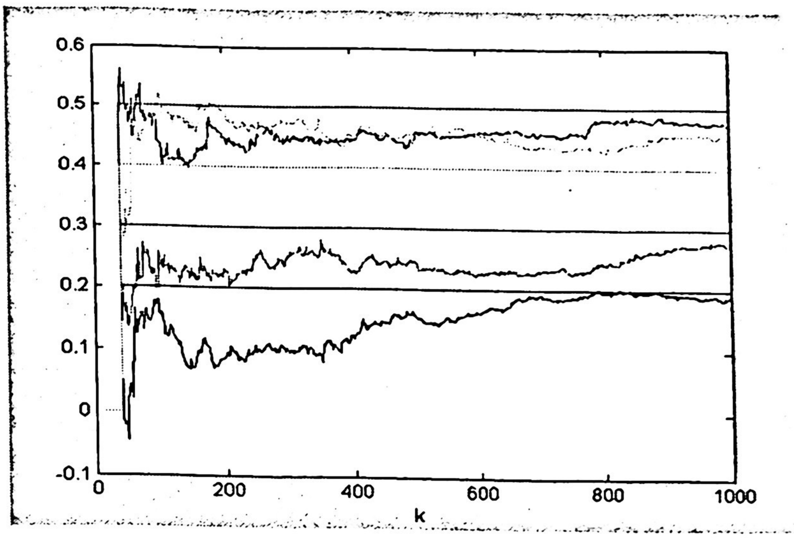

Fig.1 *Matrix parameter estimation "A" using the RTPE.*

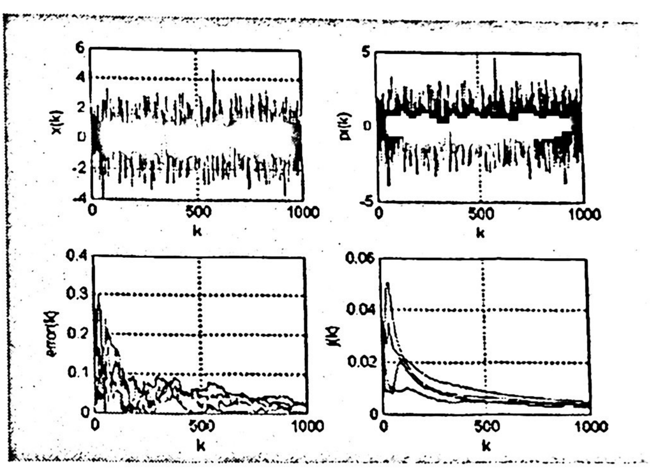

Fig. 2 *Internal state X_k, observable signal Π_k, estimation error and functional error J_k, for RTPE model.*

5 Conclusions

The internal dynamic system for Periodic Real Time Tasks into a concurrent system is into the real-time system, the start time model. Tasks set, depended on its arrival and start times in accordance with its matrix parameters, its entrance and internal computing equipment and external perturbations, generating the proposition set. The model showed capable of characterize several Real Time Task behaviors and obeyed characteristics mentioned by several authors round off the real-time systems. By the other hand, the real-time internal description considers that the task characteristics, synchronicity, sampling periods and convergence time, form part of its properties. For start time estimation, the convergence times where bounded by an Δ estimation error, described by second probability moment respect to internal and external perturbations and convergence periods acceptable in probability sense. The instrumental variable technique considered in its estimator had a good enough response in time sense obeying to Real Time System conditions, approaching to real parameters rapidly and accomplishing its deadlines.

Bibliography

[1] Baras J. (1999), "*Symbolic and numeric Real-time signal processing*", Technical Report University of Maryland, USA. pp 226

[2] Caines P. (1986). "*Linear Stochastic Systems*". Ed. Wiley, Canada.

[3] Chui C., Chen G. (1999), "*Kalman Filtering with Real-time Applications*". Ed. Springer, USA.

[4] Cruz-Pérez D. (2004). "Modelo Dinámico para Tareas en Tiempo Real". Tesis de Maestría en Ciencias de la Computación, 7 de agosto de 2004, CIC-IPN, México D.F.

[5] Ecker K. H. (2000). "*Overview on Distributed Real-time Systems. Requirements, Analysis, Specification, Operating Systems, Case Studies*". Institute of Informatik, Technical University of Clausthal, Germany.

[6] Guevara P., Medel J. J., Cruz D. (2004). "*Modelo Dinámico para una Tarea en Tiempo Real*". Revista Computación y Sistemas, ISSN 1405-5546, Vol. VIII No. 1, México, Septiembre de 2004.

[7] Haykin S. (1991). *Adaptive filter theory*. Prentice Hall information and system sciences series.

[8] Kotel'nikov V. A. (1933). "*On the transmission capacity of "ether" and wire in electrocommunications*". Izd, Red. Upr. RKKA (Moscow URSS) (Material for the first all-union conference on questions of communications), vol. 44, 1933.

[9] Medel J. J. (2002). "*Análisis de dos métodos de estimación para sistemas lineales estacionarios e invariantes en el tiempo con perturbaciones correlacionadas con el estado observable del tipo: Una entrada una salida*" Computación y sistemas volumen 6 número 1, México.

[10] Medel J., Guevara P., Flores A. "*RTMDF: Real-Time Multivariable Digital Filter*". International IEEE Workshop Signal Processing 2003, Poznan Polonia, October2003.

[11] Nyquist, H. (1928). *Certain Topics in Telegraph Transmission Theory*. USA. AIEE Transactions.

[12] Shannon C. E. (1948), *"A mathematical theory of communication"*. Bell Syst. Tech. J. vol. 27, pp. 379-423, 623-656, July-Oct.

[13] Whittaker E. T. (1915), *"On the functions which are represented by the expansion of interpolation theory"*. In Proc. Roy. Soc. Edimburgh, vol. 35, pp. 181-194.

[14] Guevara P., Medel J.J., Cruz D. (2004). Modelo Dinámico para Tiempos de Arribo de una Tarea en Tiempo Real. Revista Computación y Sistemas, ISSN 1405-5546, Vol. VIII No. 1, pags. 190-209, México, Septiembre de 2004.

[15] J.J. Medel, P. Guevara, A. Poznyak. (2004). "Real-time Multivariable Digital Filter using Matrix Forgetting Factor and Instrumental Variable". Automatic Control and Computer Sciences Vol. 38, No. 1 pages 40-53, ISSN 0132-4160, February. 2004, (ISI), Latvia.

[16] J.J. Medel, P. Guevara, A. Flores. (2004). "Caracterización de Filtros Digitales en Tiempo Real para Computadoras Digitales". Revista Computación y Sistemas, ISSN 1405-5546, Vol. VII No. 3, México.

[17] Guevara-López P. (2004) "Resumen de Tesis Doctoral". Revista Computación y Sistemas, ISSN 1405-5546, Aceptado para publicación, Septiembre de 2004.